STSM REPORT

STSM Application number: COST-STSM-BM1205-26656

STSM Grantee: Lorenzo Luigi Columbo

STSM title: Deterministic reference target in self-mixing interferometry

Home Institution: Università degli Studi di Bari, Bari (IT)

Host Institution: Institute Polytehcnique de Toulouse, Toulouse (FR)

STSM period: From 2015−05−11 to 2015−05−16

I. STSM PURPOSE

The main purpose of the STSM has been the study of using an additional target (named reference target) with known cinematic features as a mean to increase the intrinsically limited performances of real time, compact sensors based on self-mixing interferometry in semiconductor lasers. The theoretical results will be possibly tailored towards proof-of-principle experiments in vibrometry and velocimetry for biomedical applications.

II. DESCRIPTION OF THE WORK CARRIED OUT DURING THE STSM AND THE MAIN RESULTS OBTAINED

The sensor scheme is sketched as in Figure 1, where a partially transparent reference target (RT) moving with a known constant velocity v_r is inserted in the external cavity formed by the semiconductor laser and the object target (OT) that translates with unknown constant velocity $v_o < v_r$.

In the single longitudinal mode, slowly varying envelope approximations and single reflection in the external cavity, the behavior of a semiconductor laser under optical feedback from two independent targets can be described by an extended version the Lang-Kobayashi (LK) equations [1] that account for an external double cavity [2]

$$\frac{dE(t)}{dt} = \frac{1}{2}(1+i\alpha)(N(t)-1)E(t) + \frac{k_o\tau_p}{\tau_c}E(t-\tau_o)e^{-i\omega_0\tau_o} + \frac{k_r\tau_p}{\tau_c}E(t-\tau_r)e^{-i\omega_0\tau_r} \quad (1)$$

$$\frac{dN(t)}{dt} = \gamma\left(I_p - N(t)(1+|E(t)|^2)\right) \quad (2)$$

where the adimensional field E, carriers density N and pump intensity I_p are scaled as in [3] and the time t is expressed in units of the photon lifetime τ_p. Moreover, α is the linewidth enhancement factor, τ_c is the laser cavity round trip time, ω_0 is the solitary laser frequency (equal to the laser cavity resonance and the reference frequency), γ is the photon to carrier lifetime ratio. The feedback strength parameters k_i with $i = o, r$ depend on the effective fraction of the field back-reflected from OT and RT that re-enters into the laser cavity, and the reflectivities of the external targets and of the laser exit facet. The delays τ_i change in time due to the target motions so that $\tau_i = 2L_i/c = 2(L_{0,i} + v_i t)/c$ where $L_{0,i}$ represent the target initial position. Looking for CW solutions of Equations (1)-(2) we get the following expressions for the CW frequency ω_F and the associated difference ΔN between the carriers density in presence of feedback and its value in the free running laser case [4]

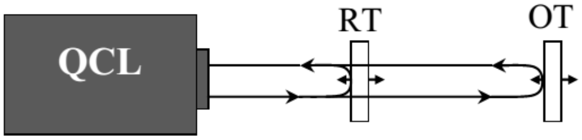

FIG. 1: Scheme of the proposed sensor

$$\begin{aligned}\omega_F &= \omega_0 - \frac{k_o\tau_p}{\tau_c}\sqrt{1+\alpha^2}\sin(A_o + \omega_o t + \arctan\alpha) \\ &- \frac{k_r\tau_p}{\tau_c}\sqrt{1+\alpha^2}\sin(A_r + \omega_r t + \arctan\alpha)\end{aligned} \quad (3)$$

$$\Delta N = -2\frac{k_o\tau_p}{\tau_c}\cos(A_o + \omega_o t) - 2\frac{k_r\tau_p}{\tau_c}\cos(A_r + \omega_r t) \quad (4)$$

τ_p	τ_c	α	ω_0	$L_{0,r}$	$L_{0,o}$
100 ps	35.6 ps	2	302 THz	1.5×10^{-3} m	2.5×10^{-3} m

TABLE I: Physical parameters used in the simulations of the LK equations

where $A_i = 2L_{0,i}\omega_F/c$ and $\omega_i = 2v_i\omega_F/c$. Hence the temporal evolution of the quantity $\Delta N = \Delta N(t)$, which is proportional to the voltage offset at the semiconductor laser terminals and thus represents one of the experimentally accessible SM signal, contains information about speed (and consequently, position) of both targets. From this follows that an explicit solution of Equations (3)-(4) would allow in principle to determine the value of the OT velocity, and thus the OT displacement. Since the highly implicit character of the transcendent Equation (3) this is analytically not possible and numerical methods have to be used to recover the information about the OT from the ΔN time trace.

To this purpose, we simulate the evolution of the SM function ΔN, by numerically solving Equations (3)-(4) for different values of v_r and v_o. The other parameters used in our simulations and reported in Table I are typical for a mid-infrared Quantum Cascade Lasers (QCL). With respect to conventional bipolar semiconductor lasers, continuous wave emission in QCLs is in fact much more stable against optical feedback provided by an external target. As we recently demonstrated this follows from the absence of relaxation oscillations due to the ultra-fast carriers recombination and to the small value of the linewidth enhancement factor (or α factor) [3, 5]. Moreover, in the last decade QCLs, that represents compact, high power, highly coherent, widely tunable laser sources in the mid-infrared to terahertz range of the electromagnetic spectrum have been extensively used in the field of medical diagnosis [6]. An example of the carrier density difference $\Delta N(t)$ obtained from our simulations is represented in Figure 2. Its temporal trace exhibits two distinct modulations, which shows the peculiar features of the lasers operating in SM configuration: slow fringes (from mark A to mark B in Figure 2a) on the scale of $\simeq$ 60 ms that modulates fast fringes (between points C and E in the close up of Figure 2b) on the scale of $\simeq 10^{-1}$ ms. While one could expect that they correspond to the slow and fast motions of the OT and RT, respectively, the inspection of the Fourier transform of $\Delta N(t)$ shown in Figure 3 reveals the nonlinear nature of the SM signal. In fact, the first peak, at frequency $\omega_o = 4\pi v_o/\lambda_0 = 100$ Hz is associated to the slow periodicity due to the movement of the OT, another peak at $\omega_r = 4\pi v_r/\lambda_0 = 10$

kHz, is associated to the motion of the RT, but the dominant peak in the spectrum occurs at $\omega_r - \omega_o$. This shows that the feedback fields provided by the targets cause a nonlinear response in the laser, due to the intrinsic nonlinearity of the LK equations (3)-(4). In the time domain, this amounts to say that there should exist temporal features of the time trace, associated to a fast time scale (being $2\pi/(\omega_r - \omega_o) \approx 2\pi/\omega_r$), which carry information about the slow time scale. Since the former is associated to the RT motion, and the latter to the OT motion, the introduction of the RT should allow to gather information about the OT motion by sampling the system at a fast rate. This overcomes the limit $\lambda/2$ for the appreciable displacement when only one target is considered.

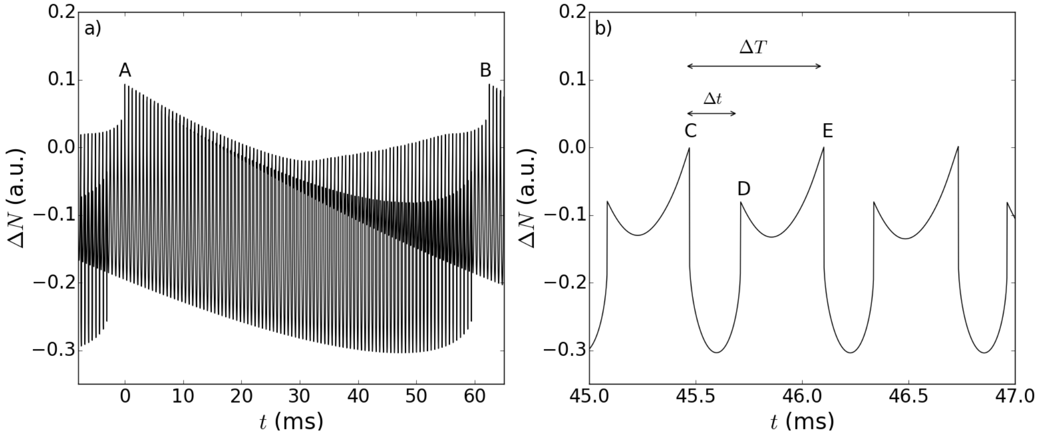

FIG. 2: Typical temporal trace of the carrier density difference ΔN as a function of time. Reference and object target velocities are $v_r = 5.0 \times 10^{-3}$ m/s and $v_o = 5.0 \times 10^{-5}$ m/s, respectively, while feedback parameters are $k_r = 0.029$ and $k_o = 0.025$. In panel a) a complete slow fringe, delimited by points A (at $t = 0$ ms) and B (at $t = 62$ ms), is shown, so that the faster modulation induced by the reference target is evident. Panel b) is a close up, for a shorter time interval, of the temporal trace shown in panel a), where the fast fringe can be identified (between points C and E), with its associated time interval, denoted with ΔT. The sub-feature is delimited by points D and E and the time interval Δt between two consecutive sub-features is the physical quantity which the sensing scheme is based on. The other parameters are reported in Table I.

More interestingly again, if the value of the feedback parameters are high enough, novel sub-features (delimited by points D and E in Figure 2b) can be seen in the time trace; such sub-features are the fingerprint of the two–target scheme and their temporal characteristics

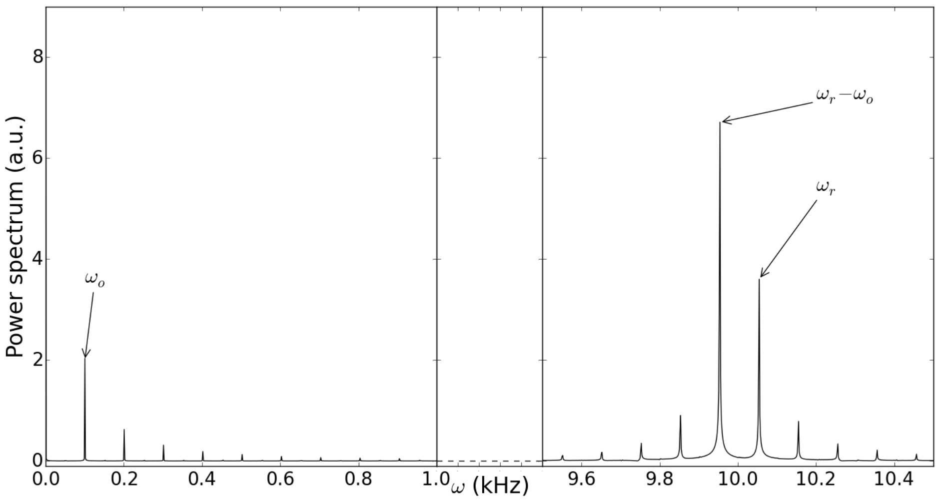

FIG. 3: Fourier transform of $\Delta N(t)$ represented in Figure 2a.

are linked to the frequency combination arising from the nonlinear response described above. The duration of these sub-features, and, conversely, the time interval (Δt) between two subsequent sub-features, also depends on the OT motion, as indicated by the fact that it varies from one fast fringe to another, within the periodicity of the slow fringe. This is precisely the temporal feature on the basis of which the slow motion of OT can be tracked in the fast fringe.

We performed an extensive fitting procedure, to relate the position of the OT to the time interval Δt between the cusps of two subsequent sub-features. In our simulations the OT velocity v_o was fixed, so we could define a "theoretical" position of the OT as given by the simple formula

$$S_{th} = v_o t. \tag{5}$$

For a set of different values of v_o and v_r, the simulated ΔN signal was sampled to extract the values Δt_n of Δt corresponding to the instants of time t_n associated with the occurrence of the left cusp (denoted as D in Figure 2b) of the $n_t h$ sub-feature, and a least square analysis was implemented to find the best fit of the relation between $S_{th}(t_n)$ and Δt_n. This procedure led us to the following interpolation formula:

$$S_{phen}(t_n) = -\gamma_2 \frac{v_r^2}{\lambda_0} \Delta t_n^2 - \gamma_1 v_r \Delta t_n + \gamma_0 \frac{\lambda_0}{2} \tag{6}$$

where the values of the constant coefficients γ_i together with their uncertainties were evaluated using all the available simulations and are given by:

$$\begin{aligned}\gamma_2 &= 1.0197 \pm 0.0017\\ \gamma_1 &= 0.6696 \pm 0.0009\\ \gamma_0 &= 0.9375 \pm 0.0002 \qquad (7)\end{aligned}$$

We can further assume that the error on the determination on the OT displacement due to uncertainties in the γ_i parameters is given by applying the error propagation law to the phenomenological formula (6)

$$\sigma_S = \sqrt{\sum_i \left(\frac{\partial S}{\partial \gamma_i}\right)^2 \sigma^2_{\gamma_i}} = \sqrt{\left(\frac{v_r^2}{\lambda_0}\Delta t_n^2\right)^2 \sigma_{\gamma_2} + (v_r \Delta t_n)^2 \sigma_{\gamma_1} + \left(\frac{\lambda_0}{2}\right)^2 \sigma_{\gamma_0}}, \qquad (8)$$

An example of the results that can be obtained with the sensing procedure described so far is given in Figure 4. It is evident by inspection of the close-up, Figure 4b, that the potential

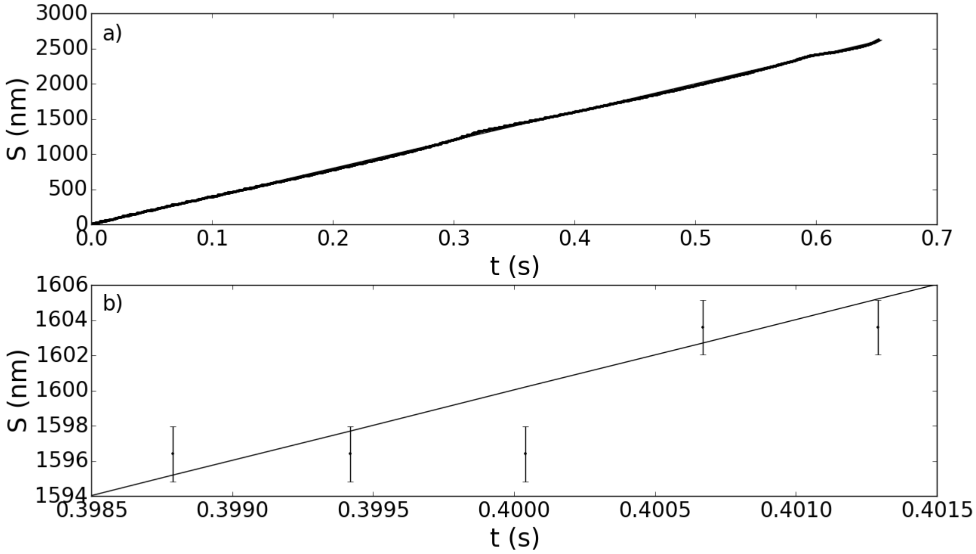

FIG. 4: Plot of the position of the OT target $S(t)$ versus time, for a simulation with $v_r = 5 \times 10^{-3}$ m/s and $v_o = 4 \times 10^{-6}$ m/s. The other parameters are as in Figure 2. The solid line represents the theoretical position S_{th} (see Equation (5)). The dots mark the position S_{phen} as predicted by the phenomenological equation (6), and the associated errors σ_S as given by Equation (8). Panel a) is the representation over the entire period of a slow fringe, while panel b) is a close–up in which the accuracy of S_{phen} in retrieving S_{th} can be appreciated.

measurements of the OT are well within the 10 nm deviation from the theoretically expected values (the solid line). More details of this numerical approach together with an experimental validation of the proposed technique are reported in [7].

In collaboration with the group of Prof. Thierry Bosch that has a consolidated experience both in the development of optical sensors based on self-mixing interferometry in the visible and near-infrared and in signal analysis, the next tasks of this research activity could be:

1. Extend these results to the case of semiconductor lasers emitting in the near-infrared or THz region of the electromagnetic spectrum. From the theoretical point of view it basically implies a change of the characteristic alpha-factor and the carriers lifetime used in the simulations.
2. Explore different regimes of feedback in order to identify the optimal operation conditions of the two-target configurations. In order to check the stability against optical feedback of the re-injected semiconductor lasers (and then the applicability of Equations (3)-(4)) we will numerically integrate the LK Equations (1)-(2) using the Matlab function *ddesd* with two time varying delays associated to the RT and OT motion.
3. Extend this technique to oscillating RT and OT, and, in perspective, to arbitrary target dynamics for possible applications to vibrometry and velocimetry. Such extension requires to study in detail the interferometric time trace from such configurations, correlating it to the spectral signatures of the nonlinear frequency mixing and eventually selecting the significant intervals to be extracted from the time trace.

In case of success we believe that this would result in a remarkable improvement of the performances (e.g. resolution) of the state of art optical techniques for biomedical imaging based on laser self-mixing in semiconductor lasers.

III. MUTUAL BENEFITS FOR THE HOME AND HOST INSTITUTIONS

The mutual benefit of the collaboration that officially started with this STSM between the theoretical group of nonlinear optics of the Università degli Studi di Bari (IT) to which the STSM Grantee belongs and the Institute Polytehcnique de Toulouse (FR) will consist for e.g. in the applications of the deterministic reference target approach to increase the resolution of self-mixing interferometry with conventional bipolar semiconductor laser in the field of velocimetry or vibrometry for biomedical applications.

IV. FUTURE COLLABORATION WITH THE HOST INSTITUTION

The STSM Grantee and Prof. Thierry Bosch plan to further collaborate on the study of self-mixing based interferometric sensors for biomedical applications.

[1] Kane, D. M.; Shore, K.A. *Unlocking dynamical diversity-Optical feedback effects on semiconductor diode lasers*, J. Wiley and Sons, 2005.

[2] Mezzapesa, F. P.; Columbo, L. L.; Dabbicco, M.; Brambilla, M.; Ancona, A.; Sibillano, T.; De Lucia, F.; Lugará, P. M. and Scamarcio, G. Simultaneous measurement of multiple target displacements by self-mixing interferometry in a single laser diode. *Opt. Express* **2011**, *19*, 16160-16173.

[3] Mezzapesa, F. P.; Columbo, L. L.; Brambilla, M.; Dabbicco, M.; Borri, S.; Vitiello, M. S.; Beere, H. E.; Ritchie, D. A. and Scamarcio, G. Intrinsic stability of quantum cascade lasers against optical feedback. *Opt. Express* **2013**, *21*,13748-13757.

[4] Mezzapesa, F. P.; Columbo, L. L.; Dabbicco, M.; Brambilla, M. and Scamarcio, G. QCL-based nonlinear sensing of independent targets dynamics. *Opt. Express* **2014**, *22*, 5867-5874.

[5] Columbo L. L.; Brambilla, M. Multimode regimes in quantum cascade lasers with optical feedback. *Opt. Express* **2014**, *22*, 10105-10118.

[6] Faist, J. *Quantum Cascade Lasers*, Oxford University Press, 2013.

[7] Mezzapesa F.; *et al.*, Nanoscale displacement sensing based on nonlinear frequency mixing in quantum cascade lasers, accepted to IEEE J. Sel. Top. Quantum Electron. (June 2015).

STSM outcome form

STSM application number	Home institution & country	Host institution & country	BM1205 WG
COST-STSM-BM1205-26656	Università degli studi di Bari,BARI (ITALY)	Institute Polytehcnique de Toulouse, TOULOUSE (FRANCE)	WGX

Objective of the collaboration

Use of a deterministic reference target in self-mixing interferometry with conventional semiconductor lasers.

Results of the collaboration

During the STSM of Dr. Lorenzo Luigi Columbo the idea of using an additional target (named reference target) with known cinematic features as a tool to increase the performances of real time, compact sensors based on self-mixing interferometry in semiconductor lasers has been explored. The theoretical results will be possibly tailored towards proof-of-principle experiments in vibrometry and velocimetry for biomedical applications.

I hereby confirm the successful execution of the STSM of DR. Lorenzo Luigi Columbo.

Yours sincerely,

PROF. Thierry Bosch

Signature:

Notes:

- **STSM application number:** please report the number assigned by the system to your application, e.g. COST-STSM-BM1205-####
- **European Network for Skin Cancer Detection using Laser Imaging WG**: please write the number of the WG within which the cooperation is meant to be established.
- **Objective of the collaboration**: you can report even the STSM title and maybe expand it a little bit to make it clearer, if necessary.
- **Results of the collaboration:** please write a very concise synthesis. We need just a few lines that convey the essence of the outcomes. You can write one or two sentences (no more), or make a list of key outcomes (just a few words for each item). Be aware that this information is meant to be presented as a record of a big table for the whole STSM programme: we donǒt need extended descriptions (as they are already reported in your STSM scientific report).